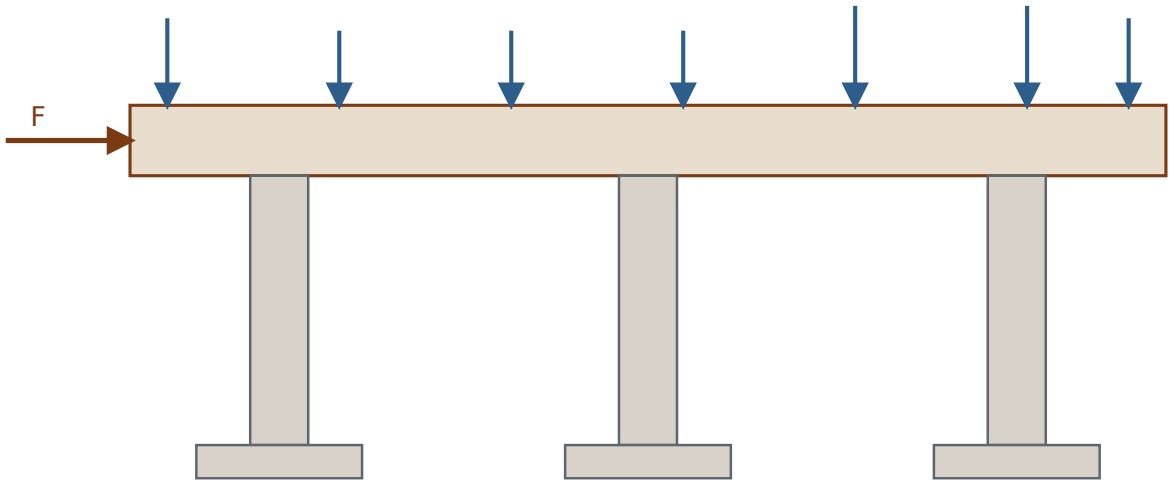
FOREWORD & HANDBOOK

Bridge Piers as Rigid Frames

Two-, three- and four-leg piers by slope-deflection, from the GW-BASIC programs of Newman E. Argraves (c. 1985–1989) to a browser page that shows its work.



cap on three legs; girder reactions above, lateral force F at cap level

Gary Argraves & Claude · November 1, 2026

In memory of Newman E. (Edward) Argraves, Connecticut State Highway Commissioner, engineer's engineer.

Foreword

Newman E. Argraves built I-95 through Connecticut as State Highway Commissioner. He had sold his stake in private practice when he took the post, so that there could be no question of a conflict of interest, and he drove a beat-up Volkswagen Karmann Ghia. Around 1985, on an Ericsson suitcase portable his grandson Gary found for him, he wrote GW-BASIC programs to solve two-, three- and four-legged bridge piers. His daughter Nancy typed the code into the machine. He kept revising them until at least February 1989.

Those programs are the method an engineer of his generation carried in his head and on paper: the pier cap and legs taken as one rigid frame, the joints and the sway solved together by slope-deflection, then the shears, moments, face values, axial loads, temperature and shrinkage worked out in order. The listings survived, along with his data and his printed runs.

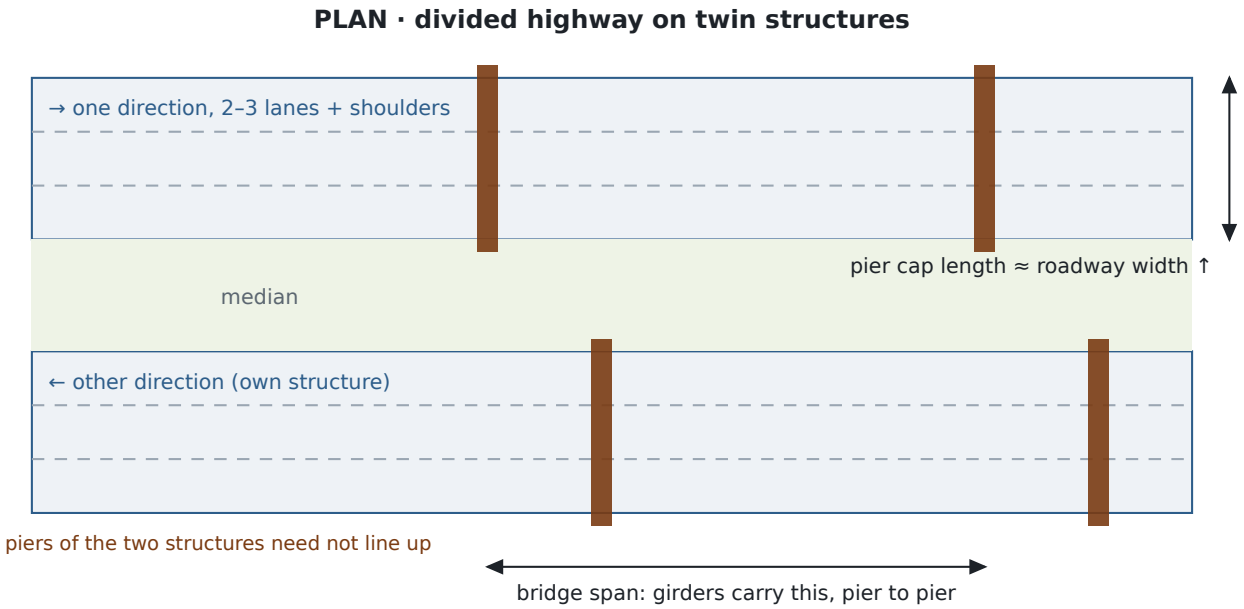
This handbook goes with `pier.html`, a single browser page that runs all three pier families from a plain-text deck, shows the work line by line as it would be done on paper, plots the frame, and checks itself against his own printed answers. Nothing is installed and nothing is hidden: his listings sit one link from every result.

The aim is a simple one. A designer who never met him should be able to open the page, understand what the numbers mean and where they come from, and trust them as far as the checks go, and no further. The past stays in use, and the work going forward is shown on the page.

> **Newman's rule.** An Army wind-tunnel design team could not decide which direction to strengthen the grounding supports, and called him. His answer: *resist the full forces in both, in all, directions.* Forces that can reverse will. He ran the same three-leg pier with F = +45.5 k and with F = −45.5 k.

1 · Where the pier sits: the spans are not the bridge spans

The first thing that looks wrong to a newcomer is the span length: 12 to 20 ft seems far too short for a highway bridge. It is short because the pier "spans" run *across* the road, between the legs under the cap. The bridge spans run *along* the road, pier to pier, and are carried by the girders. The pier only feels them as large point loads where each girder line bears on the cap.



Each one-way roadway sits on its own row of piers. Cap length ≈ roadway width; the long spans are along traffic.

Example (his data)	Cap geometry	Reads as
3 legs, PR3LDG3Z	2 × 20.5 ft between legs + 2 × 5.5 ft overhangs = 52 ft; seven girder lines at about 8.3 ft	one direction: three 12-ft lanes plus shoulders
4 legs, PR4BM4I	3 × 19.25 ≈ 58 ft between outer legs	a somewhat wider one-way deck
4 legs, PR4BMQ	3 × 12.84 ≈ 38.5 ft between outer legs	a narrower roadway or ramp
2 legs, PIER2LD	one 20.5 ft span plus overhangs	a narrow deck

Why twin structures

- **Offset and skew.** Each structure places its piers where its own ground, crossing road or stream allows. The two rows need not line up.
- **Different leg heights.** The two roadways often sit at different grades and cross-slopes. His programs take a separate H for every leg ("DIFFERENT LENGTHS" in his titles).
- **Staged work.** One structure carries both directions while the other is built or rebuilt, on a road that cannot close.
- **A manageable cap.** A 50-odd foot cap on three legs is a far easier frame than a 100-ft cap across the whole highway. The 2-, 3- and 4-leg families cover the range of one-way roadway widths.

The size of the girder reactions, 145 to 236 k on one beam line, is where the long bridge spans show up: it is that girder's share of the deck from both sides of the pier, dead plus live.

2 · The method

The cap and legs are one rigid frame. The unknowns are a rotation θ at each cap joint and one sway Δ of the cap (and, with footing springs, a rotation at each leg base). Slope-deflection gives every member end moment in terms of them:

```
M_near = 2·E·K·(2·θ_near + θ_far − 3·Δ/H) + FEM          K = I / length
```

One equation per joint (the moments meeting there sum to zero) and one for sway (the leg shears balance F) are solved together by Gaussian elimination with row swap, exactly as he programmed it. The report then works, in order: properties and stiffnesses, fixed-end moments, cantilever moments ZL and ZR, the matrix, the solution, end moments, equilibrium checks, span shears and moments with face-of-leg values, cantilever faces, leg axial loads with self-weight, the footing check, temperature and shrinkage, and the answer key.

Moments depend only on stiffness *ratios*, so E cancels out of them. E matters only where a real movement is imposed: rotations, sway, temperature and shrinkage.

3 · Reading the loads: the not-so-obvious

Point	What it means
w is the cap, not the bridge	The uniform load is the cap's own weight, BW·BD·0.150 kcf: 3 × 4 × 0.15 = 1.80 k/ft in the 3-leg example, 3 × 2.5 × 0.15 = 1.13 in the 4-leg. It is light on purpose.
The point loads are the bridge	Each P is a girder reaction: one beam line of the superstructure, dead plus live. Their positions are the girder lines. Nearly all the load on a pier comes this way.
The outer loads ride the cantilevers	The fascia girders sit on the overhangs and are usually the heaviest (parapet, sidewalk, rail). They twist the cap at the outer joints: ZL and ZR.
Unequal girder loads are deliberate	Live load is placed lane by lane for the worst case. Heavier reactions on one side (236 k against 145.6 k) unbalance the cap, rotate the joints and sway the frame.
F reverses: run it both ways	Wind on superstructure and pier, braking, stream flow: the lateral force at cap level can come from either side. In the 3-leg example, reversing F changes the outer leg-top moments from about −200 and −227 ft-k to +82 and +55. Design each member for the worse.
Legs bend both ways	Under sway a leg fixed at both ends bends in double curvature; the moment changes sign up the leg. Steel must work on both faces.
The base moment goes to the footing	A fixed leg hands its base moment to the footing with the axial load; e = M/N decides the soil pressure.
Temperature works from the centre	The cap grows and shrinks about the pier centre, so the centre leg of three does not move and the outer legs take the moment.

Shrinkage is not in the load diagrams	Shrinkage and temperature are worked apart from the loads, as he did, and can exceed the load moments in the outer legs: 323 ft-k against 227 in the 3-leg example. The page warns on the plot, flags it in the summary box, and can add shrinkage plus a temperature fall to the leg diagrams. The formula $6EI\delta/H^2$ is an upper bound (fixed legs, full E, gross I); creep and cracking reduce it, so ε and ΔT are the designer's inputs, not defaults to accept.
Sway load factor	F enters times a factor, default 1.0. His BM revision used 0.75 (his P5 written with 4·2 for 3·2). It is an input so the designer decides.

4 · Supports and footings

All his runs assume legs **fixed** at the base. Fixity is a real detail, not a label: it means dowels from the footing into the leg and a footing stiff enough, on soil stiff enough, not to turn. A **pinned** base would imply no steel carried from footing into leg. Real footings sit between the two.

The page brackets this with a rotational spring at each base, from the footing size and the soil:

```
kr = ks · Lf · B³ / 12        ks = modulus of subgrade reaction (k/ft³)
     ≈100 loose sand · 300 medium · 600+ dense sand / stiff clay
.footing B=10 Lf=12 ks=300    every leg      .leg 2 ... kr=0    one leg pinned
```

Softer soil lets the footing turn; base moments fall and the leg tops and cap take more. A very large ks reproduces the fixed result exactly; kr = 0 gives a true pin with zero base moment. The footing check then works the bearing pressure: within the kern ($e \le B/6$) $q = N/(B \cdot Lf) \cdot (1 \pm 6e/B)$, otherwise $q_{max} = 2N / (3 \cdot Lf \cdot (B/2 - e))$.

The complexity of a multi-leg frame, its sway and its footings is part of why many crossings use wall-type abutments and wall piers instead. Where open legs are wanted (clearance, sight lines, stream flow, less concrete), this is the analysis that has to be done.

Overhang loads: two ways

The overhang delivers a shear V and a moment Z to the outer joint. The deck supplies them one of two ways, one way per side.

	Case 1 · computed: `.cantL / .cantR`	**Case 2 · hand-worked: `.joint`**
Who uses it	most designs; the default	Newman's 1989 4-leg way; any overhang the program cannot describe
Deck	`.cantL LC=5.5 X=4.25 P=-201.6 w=-1.8`	`.joint ZL=-150 VL=-50` `.joint ZR=-150 VR=-50`
Moment at joint	$Z = P \cdot X + w \cdot LC^2/2$, worked every run	typed in from the overhang calculation (hogging negative)

Shear at joint	P + w·LC, worked every run	typed in (down negative)
Face values	V and M at the leg face, in the report	from the hand calculation
Good for	one bearing per overhang plus self-weight	two bearings, parapet, sidewalk, utilities; a fixed design whose loads are being iterated

His 4-leg BM revision reads no overhang data, so he worked the overhang by hand and typed the moment into lines 725 and 740: 150 / 150 in PR4BMQ (50 k × 3.0 ft), 151 / 156 in PR4BM4I. The earlier 4-leg programs computed it, as the 3-leg does. Give the shear with the moment: a moment alone leaves the overhang load out of the leg axial loads. (His runs carried it as the bearing load listed in the span at 0.2 ft.) Check hand numbers once against Case 1; for a single bearing they must agree.

5 · Conventions and the checks that must close

Quantity	Convention
Joints and legs	His letters: cap joints A, B, F, G from the left; leg bases D, C, E, H. Spans AB, BF, FG; legs AD, BC, FE, GH.
Loads	down negative, as in his DATA
End moments	clockwise positive on the member end (slope-deflection)
Cap internal moment	sagging positive; M(left end) = MAB, M(right end) = −MBA
Leg shear	V = (Mtop + Mbase)/H
Leg axial	compression positive, including leg self-weight γ·CW·D·(H − deduct)

Every run prints these, and every one must close:

```
Joint balance   Σ member end moments − ZL (left joint) / + ZR (right joint) = 0.000
Sway            Σ V + F · factor                                             = 0.000
Base springs    M_base + kr · θ_base                                         = 0.000
Spans           shear and moment at each end agree with the end moments
Vertical        Σ N = Σ loads + self-weight
```

On the plot the hover pane lists each joint's moments and shows them summing to zero. A number that does not close is a wrong number, however neat it looks.

6 · Verification record

His program	Result on the page	Remaining difference explained by
PIER2LD (2 legs)	matches every printed digit	—

PR3LDG3Z (3 legs)	within 0.11 %	his CBF leaves out MC4A4 (P9's left fixed-end moment); replicated: exact
PR4BMQ (4 legs)	within 0.20 %	his sway coefficient −2.67 for −8/3; replicated: exact
PR4BM4I (4 legs)	within 0.43 %	the same −2.67; replicated: exact
Framer, 5 legs (wind, dead, all loads)	within 0.016 ft-k	Framer applies loads as strips 0.001 ft wide, F just below the joint

He wrote programs for 2, 3 and 4 legs. The page solves 2 to 6; five legs (the multi-column type seen on I-84 at Southbury, no overhangs) are proved against Framer, a stiffness-method frame program independent of his code. Framer prints end moments counterclockwise positive, so its key enters with the sign changed. Framer takes point loads as narrow strips (W = P/C), so the deck puts F as a 0.001 ft strip ending at the cap, the way the page applies it. A first run with a 0.1 ft strip lower on the column shifted the sway moments by up to 1.6 ft-k; an independent solve with the load placed the same way matched Framer to 0.002 ft-k. New joint letters continue his pattern: J, M at the cap, K, N at the bases.

Limits: a very stiff footing equals the fixed answer; kr = 0 gives zero base moment; reversing F mirrors the sway part of the answer. All equilibrium checks close in all six example decks.

7 · Corrections to the 1989 listings

Each was proved by statics in his own runs. None touches the solve or the end moments; they are in the follow-on shears, face values and side calculations. His originals are kept unchanged beside the page.

Line(s)	As written	Corrected
`2104 (2-leg)`	`MCR … WU*(L-A1B)^2`	/2 added
`2258 (2-leg)`	`WU*(C31-A1B)`	C31−B1B: right-end shear then equals SL + WT
`4661 (2-leg)`	`WU(B1A-D/2)`	missing * (GW-BASIC read it as an array, value 0)
`2105 (3L, 3LD)`	`RL = (−ZL+MBA)/L`	(MAB+MBA)/L, as he later did in G3Z
`2300 (3L, 4L)`	`MA … + .3*ZL`	removed; his BM revision agrees
`2312 (2/3-leg)`	MRFCA uses SFC before it exists	integrated to the face
`3289 (G3Z)`	B1A in span BF	B3A
`3343 (G3Z)`	`(SA4+SFL)/2`	SA4R
`3190 (4-leg)`	`WUM*B3A`	`WUM*B3A^2/2`
`1460 (PIER4L)`	MC4ARA in CFB	removed (P9 is a right-span load)
`1375 (3-leg)`	MC4A4 computed, not added to CBF	added

`D*CW*(H-3)`	D is the sway there, H unset	each leg D·CW·(H − deduct)
`4841 (G3Z)`	shrinkage as a force	moment $6 \cdot E \cdot 144 \cdot I \cdot \delta / H^2$
2-leg temperature	one leg L, the other 0	each leg its distance from the pier centre
`2520/3520 (BM)`	max-moment search	where the shear crosses zero

Two load-model choices were made with Gary: a bearing load on an overhang is carried by the cantilever, so its shear at the leg face is fully additive (his "inspan" model is kept to reproduce his runs), and one bearing load per overhang is enough.

8 · What remains the designer's job

The page is an analysis, not a code check. Verified means: it solves the frame he solved, the way he solved it, and every equilibrium check closes. It does not choose the loads.

- **Load factors and combinations** under the governing code edition; the sway factor and the temperature range are inputs for that reason.
- **Live-load placement**: lanes loaded for the worst cap moment, the worst leg moment and the worst footing are different cases. Run each.
- **Lateral both ways**: Newman's rule. Run +F and −F.
- **Footing stiffness**: bracket it, fixed and on springs, and design for the envelope.
- **Member design**: reinforcement, shear, crack control, slenderness of the legs and second-order effects, bearing seats, and the footings themselves.

9 · The pathway: from the run to the page

The same rule governs every port in this family (concrete beam, ultimate-strength carry sheet, bridge pier):

> **The original run is the proof; the JavaScript is the port.** A FRAME `.frm` run with `frame.lua`, or a GW-BASIC run in PC-BASIC, sets the answer. The web page must reproduce it, line by line, with the work shown as on paper, and must carry the answer key so anyone can check.

```
/var/www/main/
  frame/                       browsable source (Caddy file_server browse; mkindex.sh)
    forms/  *.frm, *.out.txt, PAGES.txt     templates, runs, form-to-page map
    lua/    frame.lua                       the engine
    pascal/ frame.pas, bin/                 the original engine and Linux binary
  pier/
    pier.html                  the page: deck · Report · Plots · Instructions
    pier_core.js  pier_decks.js  pier_shell.html   build parts (awk splice)
    legacy/  *.TXT *.RUN *.IN  his listings, runs and data, kept as found
```

Every page's Instructions link to its sources in one side browser tab, so the original and the port can be compared side by side. To rerun his programs on Linux:

```
pcbasic -n -q PIER2LD.TXT > PIER2LD.RUN
```

Afterword

He solved these frames with an ordinary portable computer, a careful daughter at the keyboard and a method he could check by hand. The page keeps all of that: his letters, his names beside the new ones, his runs as the answer key, his corrections shown rather than hidden. What it adds is reach. The next designer can see the frame bend, watch every joint sum to zero, and follow any number back to 1985.

Gary Argraves & Claude · Newtown, Connecticut · November 2026